nag_sinh (s10abc)

1. Purpose

nag_sinh (s10abc) returns the value of the hyperbolic sine, $\sinh x$.

2. Specification

```
#include <nag.h>
#include <nags.h>

double nag_sinh(double x, NagError *fail)
```

3. Description

The function calculates an approximate value for the hyperbolic sine of its argument, $\sinh x$.

For $|x| \leq 1$ the function is based on a Chebyshev expansion.

For $1 < |x| \leq E_1$, (where E_1 is a machine-dependent constant), $\sinh x = \frac{1}{2}(e^x - e^{-x})$.

For $|x| > E_1$, the function fails owing to the danger of setting overflow in calculating e^x. The result returned for such calls is $\sinh(\text{sign}\, xE_1)$, i.e., it returns the result for the nearest valid argument.

4. Parameters

x
Input: the argument x of the function.

fail
The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_REAL_ARG_GT
On entry, $|\mathbf{x}|$ must not be greater than $\langle value \rangle$: $\mathbf{x} = \langle value \rangle$.
The function has been called with an argument too large in absolute magnitude. There is a danger of setting overflow. The result is the value of sinh at the closest argument for which a valid call could be made. (See Section 3 and the Users' Note for your implementation).

6. Further Comments

6.1. Accuracy

If δ and ϵ are the relative errors in the argument and result, respectively, then in principle

$$|\epsilon| \simeq |x \coth x\delta|.$$

That is, the relative error in the argument, x, is amplified by a factor, approximately $x \coth x$. The equality should hold if δ is greater than the ***machine precision*** (δ is a result of data errors etc.), but if δ is simply a result of round-off in the machine representation of x, then it is possible that an extra figure may be lost in internal calculation round-off.

It should be noted that for $|x| \geq 2$

$$\epsilon \sim x\delta = \Delta$$

where Δ is the absolute error in the argument.

6.2. References

Abramowitz M and Stegun I A (1968) *Handbook of Mathematical Functions* Dover Publications, New York ch 4.5 p 83.

7. See Also

None.

8. Example

The following program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

8.1. Program Text

```
/* nag_sinh(s10abc) Example Program
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>


main()
{
  double x, y;

  /* Skip heading in data file */
  Vscanf("%*[^\n]");
  Vprintf("s10abc Example Program Results\n");
  Vprintf("     x           y\n");
  while (scanf("%lf", &x) != EOF)
    {
      y = s10abc(x, NAGERR_DEFAULT);
      Vprintf("%12.3e%12.3e\n", x, y);
    }
  exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
s10abc Example Program Data
           -10.0
            -0.5
             0.0
             0.5
            25.0
```

8.3. Program Results

```
s10abc Example Program Results
     x           y
  -1.000e+01  -1.101e+04
  -5.000e-01  -5.211e-01
   0.000e+00   0.000e+00
   5.000e-01   5.211e-01
   2.500e+01   3.600e+10
```