nag_cos_integral (s13acc)

1. Purpose

nag_cos_integral (s13acc) returns the value of the cosine integral $\mathrm{Ci}(x)$.

2. Specification

```
#include <nag.h>
#include <nags.h>

double nag_cos_integral(double x, NagError *fail)
```

3. Description

The function evaluates

$$\mathrm{Ci}(x) = \gamma + \ln x + \int_0^x \frac{\cos u - 1}{u}\,du \qquad x > 0$$

where γ denotes Euler's constant.

The approximation is based on several Chebyshev expansions.

4. Parameters

x

Input: the argument x of the function.
Constraint: **x**> 0.0.

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_REAL_ARG_LE

On entry, **x** must not be less than or equal to 0.0: **x** $= \langle$*value*$\rangle$.

The function is not defined for this value and the result returned is zero.

6. Further Comments

6.1. Accuracy

If E and ϵ are the absolute and relative errors in the result and δ is the relative error in the argument then in principle these are related by $|E| \simeq |\delta \cos x|$ and $|\epsilon| \simeq |(\delta \cos x)/\mathrm{Ci}(x)|$. That is, accuracy will be limited by ***machine precision*** near the origin and near the zeros of $\cos x$, but near the zeros of $\mathrm{Ci}(x)$ only absolute accuracy can be maintained.

For large values of x, $\mathrm{Ci}(x) \sim (\sin x)/x$ therefore $\sim \delta\, x \cot x$ and since δ is limited by the finite precision of the machine it becomes impossible to return results which have any relative accuracy. That is, when $x \geq 1/\delta$ we have that $|\mathrm{Ci}(x)| \leq 1/x \sim E$ and hence is not significantly different from zero.

Hence, for $x > x_{\mathrm{hi}}$, where x_{hi} is a machine-dependent value, $\mathrm{Ci}(x)$ in principle has values less than ***machine precision***, and so is set directly to zero.

6.2. References

Abramowitz M and Stegun I A (1968) *Handbook of Mathematical Functions* Dover Publications, New York ch 5.2 p 231.

7. See Also

nag_sin_integral (s13adc)

8. Example

The following program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

8.1. Program Text

```
/* nag_cos_integral(s13acc) Example Program
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

main()
{
  double x, y;

  /* Skip heading in data file */
  Vscanf("%*[^\n]");
  Vprintf("s13acc Example Program Results\n");
  Vprintf("     x           y\n");
  while (scanf("%lf", &x) != EOF)
    {
      y = s13acc(x, NAGERR_DEFAULT);
      Vprintf("%12.3e%12.3e\n", x, y);
    }
  exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
s13acc Example Program Data
             0.2
             0.4
             0.6
             0.8
             1.0
```

8.3. Program Results

```
s13acc Example Program Results
     x           y
   2.000e-01  -1.042e+00
   4.000e-01  -3.788e-01
   6.000e-01  -2.227e-02
   8.000e-01   1.983e-01
   1.000e+00   3.374e-01
```