# nag_kelvin_bei (s19abc)

## 1. Purpose

**nag_kelvin_bei (s19abc)** returns a value for the Kelvin function bei $x$.

## 2. Specification

```
#include <nag.h>
#include <nags.h>

double nag_kelvin_bei(double x, NagError *fail)
```

## 3. Description

This function evaluates an approximation to the Kelvin function bei $x$.

The function is based on several Chebyshev expansions.

For large $x$, there is a danger of the result being totally inaccurate, as the error amplification factor grows in an essentially exponential manner; therefore the function must fail.

## 4. Parameters

**x**
Input: the argument $x$ of the function.

**fail**
The NAG error parameter, see the Essential Introduction to the NAG C Library.

## 5. Error Indications and Warnings

**NE_REAL_ARG_GT**
On entry, $|\mathbf{x}|$ must not be greater than $\langle value \rangle$: $\mathbf{x} = \langle value \rangle$.
$|\mathbf{x}|$ is too large for an accurate result to be returned and the function returns zero.

## 6. Further Comments

### 6.1. Accuracy

Since the function is oscillatory, the absolute error rather than the relative error is important. Let $E$ be the absolute error in the function, and $\delta$ be the relative error in the argument. If $\delta$ is somewhat larger than the ***machine precision***, then we have $E \simeq |x(-\mathrm{ber}_1 x + \mathrm{bei}_1 x)/\sqrt{2}|\,\delta$ (provided $E$ is within machine bounds).

For small $x$ the error amplification is insignificant and thus the absolute error is effectively bounded by the ***machine precision***.

For medium and large $x$, the error behaviour is oscillatory and its amplitude grows like $\sqrt{x/2\pi}e^{x/\sqrt{2}}$. Therefore it is impossible to calculate the functions with any accuracy when $\sqrt{x}e^{x/\sqrt{2}} > \sqrt{2\pi}/\delta$. Note that this value of $x$ is much smaller than the minimum value of $x$ for which the function overflows.

### 6.2. References

Abramowitz M and Stegun I A (1968) *Handbook of Mathematical Functions* Dover Publications, New York ch 9.9 p 379.

## 7. See Also

nag_kelvin_ber (s19aac)
nag_kelvin_ker (s19acc)
nag_kelvin_kei (s19adc)

## 8. Example

The following program reads values of the argument $x$ from a file, evaluates the function at each value of $x$ and prints the results.

### 8.1. Program Text

```
/* nag_kelvin_bei(s19abc) Example Program
 *
 * Copyright 1996 Numerical Algorithms Group.
 *
 * Mark 4, 1996.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

main()
{
  double x, y;

  /* Skip heading in data file */
  Vscanf("%*[^\n]");
  Vprintf("s19abc Example Program Results\n\n");
  Vprintf("     x           y\n\n");
  while (scanf("%lf", &x) != EOF)
    {
      y = s19abc(x, NAGERR_DEFAULT);
      Vprintf("%12.3e%12.3e\n", x, y);
    }
  exit(EXIT_SUCCESS);
}
```

### 8.2. Program Data

```
s19abc Example Program Data
             0.1
             1.0
             2.5
             5.0
             10.0
             15.0
             -1.0
```

### 8.3. Program Results

```
s19abc Example Program Results

     x           y

   1.000e-01   2.500e-03
   1.000e+00   2.496e-01
   2.500e+00   1.457e+00
   5.000e+00   1.160e-01
   1.000e+01   5.637e+01
   1.500e+01  -2.953e+03
  -1.000e+00   2.496e-01
```