nag_kelvin_ker (s19acc)

1. Purpose

nag_kelvin_ker (s19acc) returns a value for the Kelvin function $\ker x$.

2. Specification

```
#include <nag.h>
#include <nags.h>

double nag_kelvin_ker(double x, NagError *fail)
```

3. Description

This function evaluates an approximation to the Kelvin function $\ker x$.

The function is based on several Chebyshev expansions.

For large x, $\ker x$ is so small that it cannot be computed without underflow and the function evaluation fails.

4. Parameters

x

Input: the argument x of the function.
Constraint: $\mathbf{x} > 0$.

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_REAL_ARG_GT

On entry, **x** must not be greater than $\langle$*value*$\rangle$: $\mathbf{x} = \langle$*value*$\rangle$.
x is too large, the result underflows and the function returns zero.

NE_REAL_ARG_LE

On entry, **x** must not be less than or equal to 0.0: $\mathbf{x} = \langle$*value*$\rangle$.
The function is undefined and returns zero.

6. Further Comments

Underflow may occur for a few values of x close to the zeros of $\ker x$, which causes a failure **NE_REAL_ARG_GT**.

6.1. Accuracy

Let E be the absolute error in the result, ϵ be the relative error in the result and δ be the relative error in the argument. If δ is somewhat larger than the ***machine precision***, then we have $E \simeq |x(\mathrm{ker}_1 x + \mathrm{kei}_1 x)/\sqrt{2}|\,\delta$, $\epsilon \simeq |x(\mathrm{ker}_1 x + \mathrm{kei}_1 x)/\sqrt{2}\mathrm{ker} x|\,\delta$.

For very small x, the relative error amplification factor is approximately given by $1/|\log x|$, which implies a strong attenuation of relative error. However, ϵ in general cannot be less than the ***machine precision***.

For small x, errors are damped by the function and hence are limited by the ***machine precision***.

For medium and large x, the error behaviour, like the function itself, is oscillatory, and hence only the absolute accuracy for the function can be maintained. For this range of x, the amplitude of the absolute error decays like $\sqrt{\pi x/2}e^{-x/\sqrt{2}}$ which implies a strong attenuation of error. Eventually, $\ker x$, which asymptotically behaves like $\sqrt{\pi/2x}e^{-x/\sqrt{2}}$, becomes so small that it cannot be calculated without causing underflow, and the function returns zero. Note that for large x the errors are dominated by those of the **math library** function exp.

6.2. References

Abramowitz M and Stegun I A (1968) *Handbook of Mathematical Functions* Dover Publications, New York ch 9.9 p 379.

7. See Also

nag_kelvin_ber (s19aac)
nag_kelvin_bei (s19abc)
nag_kelvin_kei (s19adc)

8. Example

The following program reads values of the argument x from a file, evaluates the function at each value of x and prints the results.

8.1. Program Text

```
/* nag_kelvin_ker(s19acc) Example Program
 *
 * Copyright 1990 Numerical Algorithms Group.
 *
 * Mark 2 revised, 1992.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nags.h>

main()
{
  double x, y;

  /* Skip heading in data file */
  Vscanf("%*[^\n]");
  Vprintf("s19acc Example Program Results\n");
  Vprintf("     x           y\n");
  while (scanf("%lf", &x) != EOF)
    {
      y = s19acc(x, NAGERR_DEFAULT);
      Vprintf("%12.3e%12.3e\n", x, y);
    }
  exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
s19acc Example Program Data
             0.1
             1.0
             2.5
             5.0
             10.0
             15.0
```

8.3. Program Results

```
s19acc Example Program Results
     x           y
   1.000e-01   2.420e+00
   1.000e+00   2.867e-01
   2.500e+00  -6.969e-02
   5.000e+00  -1.151e-02
   1.000e+01   1.295e-04
   1.500e+01  -1.514e-08
```